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Erosion Parameters for Cohesive Sediment in Auckland Streams

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Erosion Parameters for Cohesive Sediment in Auckland Streams

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Prepared for
Auckland Regional Council

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Contents

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1 Executive Summary

This report reviews the international literature on cohesive sediment erosion and compares this with the results of Auckland NIWA experiments (Elliott et al. 2005; Debnath et al. 2007). An erosion formula(e) and a critical shear stress at which erosion begins to occur in cohesive Auckland streams is presented here. It is intended that the critical shear stress and erosion formulae will be used in subsequent studies to assess and compare potential erosion under different development and hydrological scenarios and to determine the volumetric requirements of storm-water retention structures.

The literature review of erosion in cohesive soils indicated that for compacted clay soils, the erosion rate is linearly related to excess shear stress. Departures from this relationship occur when soil properties vary with depth or when the characteristic of the soil surface changes, such as by armouring. The form of the relationships between shear stress and erosion rates in the NIWA Auckland study (Elliott et al. 2005; Debnath et al. 2007) was variable, but tended to support a linear relationship. The NIWA studies also showed erosion rates per N/m^2 varying from a few millimetres per hour to about 0.5 m per hour depending on the cohesive strength of the soil. These erosion rates are consistent with other reported studies. The critical shear stress of Auckland soils as measured by jet tests (Elliott et al. 2005) were also consistent with reported studies, but were not consistent with the flume tests (Debnath et al. 2007) that had low or zero critical shear stresses. The NIWA studies showed that the critical shear and erosion rates were very variable because of the different soil structures, particle size, mineralogy, and degree of consolidation. However, channel sizes will have adjusted to natural variations in soil properties, with wide sections where the soil is weak and narrow and/or steep sections where the soil strength is high. Thus, because the channel characteristics will already vary with soil strength, it is possible to use an average value of critical shear stress and average cross-section shape for design purposes.

The method of estimating shear stress in streams is as important as the erosion equations. We recommend that relationships between total shear stress and flow be derived from measured relationships between flow and stage. If uniform flow equations are used to predict water levels, then some variation of Manning's n with flow should be incorporated in the analysis.

We recommend that the erosion equation for cohesive Auckland streams follows that most commonly used in modelling studies $E = M_3 (\tau - \tau_c)$. The critical shear stress (τ_c) and erosion rate coefficient (M_3) will be the same for pre- and post-development conditions. An assessment of erosion rates resulting from urbanisation should take account of both the erosion threshold (τ_c) and the extent to which this threshold is exceeded. We recommend that site specific studies could be carried out to determine relationships between soil properties, as determined by the relatively simple jet tests, channel morphology, bank vegetation, and total shear stress at the channel forming (bank full) discharge. In particular, shear stress/flow relationships can be calculated for stable stream reaches and shear stresses during past high-flow events, such as the channel forming (bank full) discharge, used as a guide to the critical shear stress (eg, Julian and Torres 2006). If specific parameters are not developed for a stream, we suggest using the median critical shear stress (approximately 33 N/m^2) and a value of 0.005-0.01 $kg/m^2/s$ for the coefficient M_3.

2 Introduction

2.1 Brief

The Auckland Regional Council (ARC) has requested NIWA to carry out an assessment of relationships between stream channel erosion and flow parameters further to the work carried out by NIWA (Elliott et al. 2005). The purpose of this report is to develop an erosion formula(e) and a critical shear stress at which erosion begins to occur in cohesive Auckland streams. It is intended that the critical shear stress and erosion formulae will be used in subsequent studies to assess and compare potential erosion under different development and hydrological scenarios and to determine the volumetric requirements of storm-water retention structures.

2.2 Theory

The flow of water down a stream channel has the ability to entrain sediments from the bed and banks. Erosion, transport, and deposition of this sediment reshape the channel. These processes are complex, and formulae for the quantitative determination of the transport of sediments are usually based on experimental results in limited and simple cases. Such formulae are of great value, but must be applied within hydraulic conditions similar to those used for their derivation. Sediment transport research has been concentrated on non-cohesive sediments (sands, gravels etc.), with less focus on cohesive sediments (silts and clays). However, most generally accepted forms of the transport formulae for cohesive and non-cohesive sediments follow the classical Du Boys equation and involve the difference, or the ratio, between actual forces on the bed and a critical velocity (either shear or near bed) v_c, discharge q_c or shear stress τ_c below which no transport occurs (Henderson 1966). These formulae can be expressed as:

$$q_B = f(\tau_0 - \tau_c),\ q_B = f(v_0 - v_c), \text{ or } q_B = f(q_0 - q_c)$$

where q_B is the transport rate with a shear velocity of τ_0, water velocity of v_0 or discharge q_0 and critical values (velocity v_c, discharge q_c, shear stress τ_c) below which no transport occurs. The function f is often complex and predicts how the transport rate q_B increases as the velocity, discharge or shear stress increases above the critical value.

2.3 Report method and purpose

The first steps in estimating an erosion rate for any stream are to determine the critical value for the hydraulic parameter below which no significant sediment transport occurs, and then determine the form of the complex function predicting sediment transport. This includes deciding on the most appropriate hydraulic parameter (ie, the relative merits of using mean velocity, near bed velocity, shear velocity, discharge/unit width, and shear stress to assess erosion) and then determining the functional form of the equation, particularly whether erosion increases linearly with discharge or to some power. These steps and issues are addressed specifically in this report.

Once a sediment transport formulae is selected, there are practical difficulties involved in applying it to streams that may have different sediment and morphological characteristics and in selecting a representative channel cross-section, roughness and slope so that the hydraulic conditions (velocity or shear stress) can be predicted accurately. This stage of the erosion assessment is discussed in the section of the report under the heading "Calculation of shear stress in streams".

This report:

- reviews sediment transport formulae and methods for cohesive soils in the scientific and engineering literature, particularly the critical "no transport" values and the functional form of the equations;
- interprets the results of recent NIWA studies in light of the literature review; and finally
- suggests a sediment transport formula that can be used in Auckland streams.

3 Literature Review

3.1 Hydraulic parameters and critical velocity/shear stress

The concept that there is a critical velocity or shear stress in cohesive sediments is widely accepted in hydraulic texts (eg, Ven Te Chow 1959, Raudkivi 1990, Graf 1998), although some (eg, Lavelle et al. 1984) have argued that there is no threshold. The fact that canals are successfully constructed and operated in cohesive soils using maximum allowable velocity/shear stress design parameters (see examples in Ven Te Chow 1959, Raudkivi 1990, and Graf 1998) demonstrates conclusively that there is a maximum allowable velocity or shear stress under which canals can be operated without any apparent erosion of the bed and banks. The conflicting views probably arise because researchers are considering different types of sediments for different purposes. Much research into cohesive sediment is for marine, estuarine, or lacustrine environments, where sediments can be unconsolidated. Some of these sediments can be relatively "fresh", unconsolidated sediments with low densities that the slightest current can disturb. For example, Thorn and Parsons (1980) studied estuarine sediments and developed relationships between critical shear stress and sediment dry density and linear relationships between excess shear stress and erosion rate. However, the dry bulk density of the estuarine sediments was about 100 kg/m^3, which is almost a tenth of the dry density of the material making up the banks of Auckland streams (Elliott et al. 2005). The sediments of concern in the stream erosion process are the consolidated sediments that make up the banks and underlie the stream bed rather than the more recent and unconsolidated deposits that often lie on the surface of the stream bed and are transported and deposited in small floods and freshes.

Early formulae for erosion were often based on mean or near-bed water velocity, and Russian literature is often based on velocities. However, more recent sediment transport formulae tend to be based on shear stress. In principle the total shear stress on the bed of a stream is a simple concept (first introduced in 1754) and easily calculated. The total shear stress is the average stress over the bed of a stream (τ, units of N/m^2) that resists the gravitational forces on the water under uniform conditions.

The gravitational force is the weight of water acting down the slope of the water. From a balance between shear stress and gravitational force:

$$\tau = \gamma RS$$

where R is the hydraulic radius (units of m), γ is the specific weight of water (units of N m^{-3}) and S is the water surface slope for uniform flow (dimensionless). The specific weight of water is $\gamma = \rho g$, where ρ is the density of water (999.1 kg m^{-3}) and g is the acceleration due to gravity (9.81 m s^{-2}). This definition is the one that is usually used on open channels. An alternative definition for shear stress, used in fluid mechanics, is:

shear stress is viscosity μ times the rate of change of velocity, v , with distance from the bed, x. That is,

$$\tau = -\mu \frac{dv}{dx}$$

This definition is used in pipes and similar closed conduits and is rarely used in streams because the velocity profile is so variable and difficult to measure.

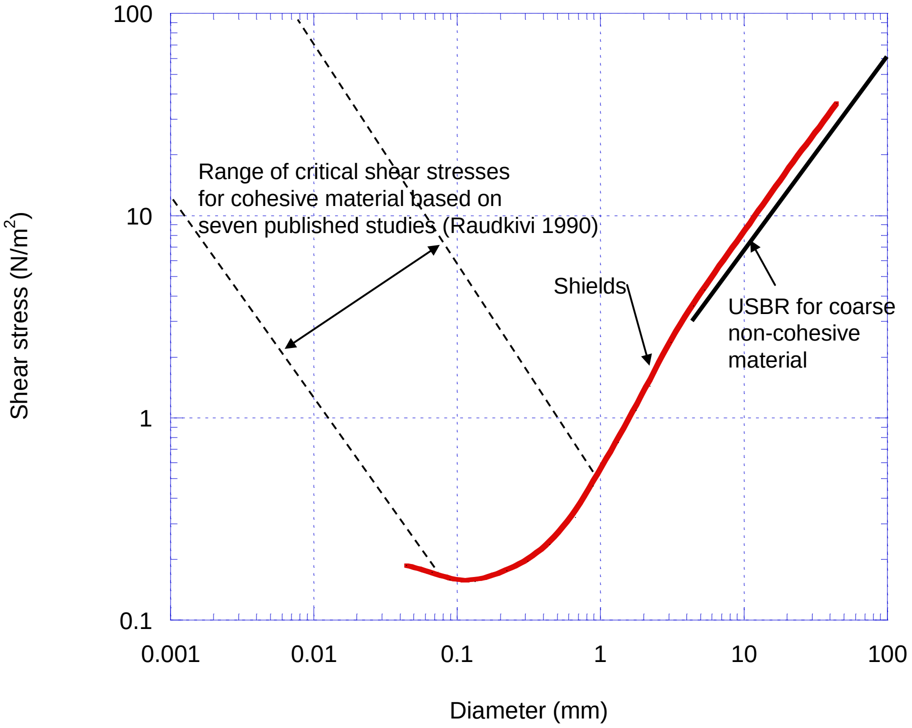
The best known examples of the shear stress and velocity approaches to initiation of movement are the Shields' diagram (Figure 1) that plots the zone of erosion (critical shear stress) against the dimensionless shear stress and dimensionless particle size and the Hjulstrom diagram (Figure 2) which plots the zone of potential erosion (critical velocity) against the particle size and velocity. Shields' diagram shows that the dimensionless shear stress required to move coarse non-cohesive particles (> 5 mm, for specific gravity 2.65) is 0.05. Shields' diagram can be expressed with dimensioned axes for water with a viscosity of 1.186 x 10^{-6} m^2/s and sediment with a specific gravity of 2.65 (Figure 1). In non-cohesive sediments, critical shear stress tends to increase as particle size increases, whereas the opposite occurs with cohesive sediments. Generally, the critical shear stress of cohesive sediments tends to be proportional to particle size to the power of -1 (Raudkivi 1990), as shown in Figure 1. The cohesive strength of soils is closely related to the clay content, with critical shear stresses increasing almost linearly with clay content, although there does not appear to be any unique relationship (Raudkivi 1990).

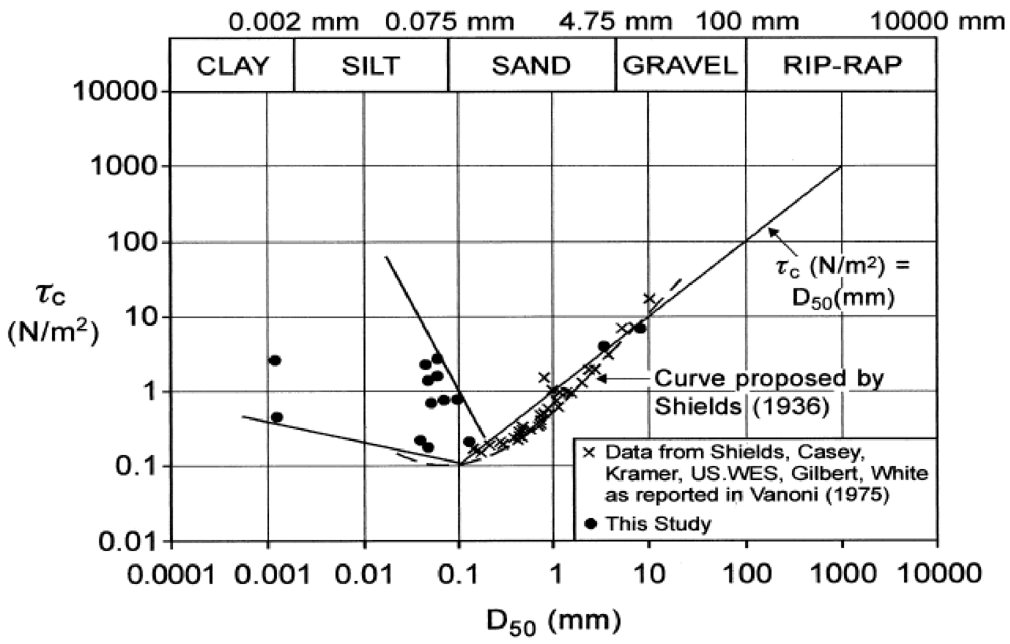
Shield's critical shear stress tends to be used in sediment transport studies and hydraulic design more than critical velocities such as in the Hjulstrom diagram, presumably because the shear stress is the force per unit bed area that water exerts on the stream bed. The unit of shear stress is a Newton per square metre (N/m^2)[1]. However, critical velocities are easier to visualise and are used as design criteria in the U.S. (Bureau of Reclamation 1977; Ven Te Chow 1959) and U.S.S.R. (Ven Te Chow 1959).

[1] 1 N/m^2 is 0.1 kg force per m^2 or 0.02 pounds force per ft^2.

Figure 1

Shields' diagram (top) of shear stress versus particle size for water and sediment with a specific gravity of 2.65, extended for cohesive soil results cited in Raudkivi (1990) and a similar diagram (bottom) produced by the team investigating the New Orleans Levee failures (University of California 2006).





In practice, shear stress is difficult to calculate because the water surface slope or energy slope varies across and along the reach of a river. The shear stress axis in Shields' diagram (Figure 1) can be converted to velocity using Manning's equation and Strickler's equation for non-cohesive particles, as follows:

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$ Manning's equation

$$n = 0.04145 d^{1/6}$$ Strickler's equation

where *v* is the velocity (m/s), *n* Manning's coefficient, and *d* the particle size (m). Rearranging Manning's equation

$$RS = \left(\frac{vn}{R^{1/6}} \right)^2$$ and substituting *RS* in the formula for shear stress $\tau = \gamma RS$ gives

$$\tau = \gamma \left(\frac{vn}{R^{1/6}} \right)^2$$

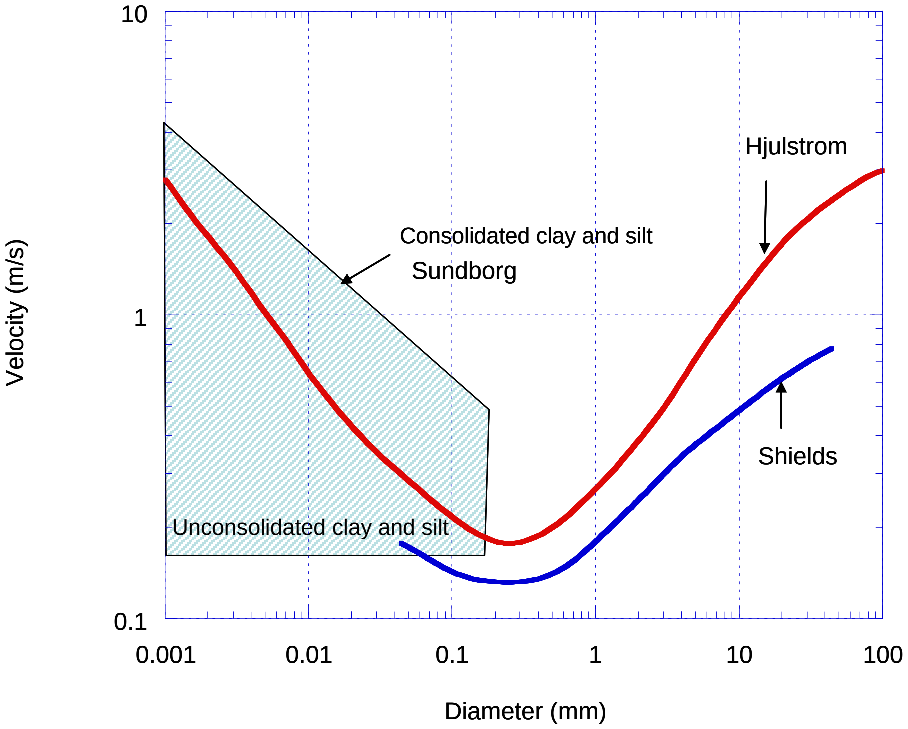
If *n* is replaced with Strickler's equation, the equation can be rearranged to give the relationship between velocity and shear stress.

$$v = \sqrt{\frac{\tau}{\gamma}} \times \frac{R^{1/6}}{0.04145 d^{1/6}}$$

The Hjulstrom and Shields' diagrams are compared in Figure 2 for water and a hydraulic radius of 1 m and particles with a specific gravity of 2.65. Both relationships show that the relationship between critical velocity and particle size changes at a particle size of about 0.3 mm. This is because transport processes of fine-grained, cohesive sediments (< 0.3 mm) are significantly different from those of coarse-grained, non-cohesive sediments, such as sands and gravels. The main difference is in the way the particles interact. Fine-grained sediment particles in the silt and clay size classes have a tendency to form agglomerations of particles called flocs, whereas coarse-grained particles in the sand and gravel size classes behave as individual particles. The velocity or shear stress required to erode cohesive sediment (d_{50}< 0.1 mm) increases as the sediment size decreases (at least until the minimum size in Shields' diagram) and that the minimum velocity for transport is about 0.15 m/s or a minimum shear stress of 0.155 N/m^2. Sundborg (1956) examined the relationship between critical velocity and particle size for a Swedish river and found that for consolidated clay and silt, the relationship was the same as that described by Hjulstrom, but for unconsolidated silt and clay, the critical velocity was constant at about 0.15 m/s for particle sizes of 0.1 to 0.001 mm (Figure 2). The strength of the agglomeration or flocculation of cohesive sediment depends on a number of factors such as particle mineralogy, the electrochemical nature of the flowing medium, and biological factors such as bacteria and other organic material and this is one reason for the large degree of variability of critical shear stress in cohesive soils. The degree of consolidation also affects strength (Figure 2). Unconsolidated fine sediments, such as recent deposits on a stream bed or estuary, are far weaker than older sediments that have been consolidated by the pressure of overlying sediments.

Figure 2

Comparison of critical velocities from Hjulstrom and critical velocities calculated from Shields' dimensionless shear stress/particle size relationship for a hydraulic radius of 1 m.



3.1.1 Use of critical velocity/shear stress in engineering design

As noted earlier, canals have been successfully constructed and operated in both non-cohesive and cohesive soils. This experience has allowed engineers to develop tables specifying maximum allowable velocity/shear stress design parameters, and these are presented in Ven Te Chow (1959). For non-cohesive soils, the allowable velocity for fine substrate (silt) is 0.15 m/s, with the allowable velocity increasing as particle size increases. This is in accordance with the Hjulstrom and Shields' diagrams shown in Figure 2 above. For cohesive soils, U.S.S.R. tables show that the maximum permissible velocity increases from about 0.3 m/s to 1.5 m/s, with allowable velocity increasing with soil compaction. These velocities are also consistent with those shown in Figure 2.

Hughes (1980) calculated velocities and depths associated with scour at 150 locations in small natural channels, and produced diagrams showing the likelihood of scour depending on velocity and water depth. For clay soils, scour was possible (1 per cent chance) when velocities exceeded 0.6 to 1.5 m/s and shear stress exceeded 10 to 20 N/m^2. Critical velocities and shear stresses were slightly lower and more variable for silty and sandy clays. Derived scour velocities associated with flow depths of between 0.15 m and 1.5 m ranged from 0.55 m/s to 0.88 m/s in sandy-silt and silty-clay soils, and from 0.67 m/s to 1.4 m/s in clay soils. Hughes considered that these values were similar to accepted maximum permissible velocity values for comparable soil conditions, and indicated that published maximum permissible velocity values were appropriate for variable intermittent flow situations as well as long-term constant flow conditions.

Ven Te Chow (1959) notes that, in cohesive soils, a deeper channel will convey water at a higher mean velocity without erosion than a shallower one, and surmises that this is probably because near bed velocities will be higher in the shallower channels than a deep channel when the mean velocities are the same. Thus, there are tables (from U.S.S.R.) that specify an adjustment that increases the allowable velocity for depth. Ven Te Chow also notes that these velocities are for straight channels, and that with sinuous channels, velocities should be reduced by 5 per cent to 25 per cent, depending upon the sinuosity of the channel.

Maximum allowable shear stress (tractive force) is also used to design stable channels, and values of allowable shear stresses are suggested for non-cohesive and cohesive soils. For coarse non-cohesive particles, the U.S. Bureau of Reclamation (U.S.B.R.) allowable shear stress in N/m^2 is 0.07 times the particle size (usually d_{75}) in mm (Henderson 1966). For fine non-cohesive soils (0.1 mm fine sand), the U.S.B.R. allowable shear stress varies from 1.2 to 3.8 N/m^2 depending on the fine sediment content of the water. In contrast, U.S.S.R. allowable shear stresses for cohesive soils are considerably higher, varying from 31 N/m^2 for compacted clays to 1 N/m^2 for loose clays (Ven Te Chow 1959).

For cohesive sediments, the determination of critical values of velocity or shear stress is regarded as a difficult task (Graf 1998). Raudkivi (1990) makes it clear that although critical shear stress and velocity are related to particle size and clay content, there is no unique relationship for all cohesive soils and that any relationship will be specific to a particular location and soil type. In addition, vegetation on the banks and roots in the soil can increase the erosive strength of soils.

Table 1 shows the range of critical shear stresses for cohesive soils reported in various publications. One problem associated with measuring critical shear stress is determining the initiation of scour. When the particles are visible to the naked eye, it is simple to detect when the first particle is scoured away. For clays this is not the case, and various investigators define the initiation of scour through different means; these vary from “when the water becomes muddy” to extrapolation of the scour rate versus shear stress curve back to zero scour rate. The lack of a precise definition for the initiation of scour may be in part responsible for the wide range of values.

Table 1

Measured critical shear stress in cohesive soils (from University of California 2006).

Author(s)	Range of τ_c (N/m^2)
Dunn (1959)	2–25
Enger et al. (1968)	15–100
Hydrotechnical Construction Moscow	1–20
Lyle and Smerdon (1965)	0.35–2.25
Smerdon and Beasley (1959)	0.75–5
Arulanandan et al. (1975)	0.1–4
Arulanandan (1975)	0.2–2.7
Kelly and Gularte (1981)	0.02–0.4

3.1.2 Critical shear stress in Auckland streams

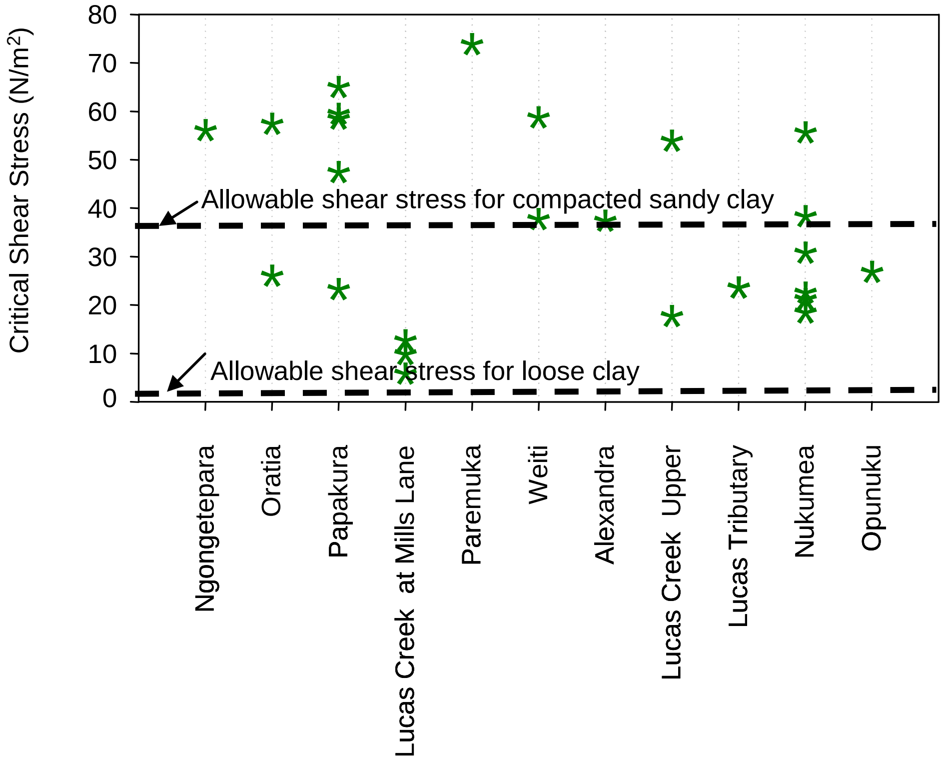
Elliott et al. (2005) measured bed and bank material characteristics at five locations in Auckland streams (Oratia, lower Lucas, upper Lucas, Nukumea, and Papakura). In these, the clay content varied from 6 to 31 per cent, the sand content was less than 50 per cent, and the bed and bank median particle size (d_{50}) varied from 0.008 mm to 0.08 mm. Thus according to Raudkivi (1990), the bed and banks of four of the five streams could be considered cohesive because they contained more than 10 per cent clay.

A series of measurements was carried out by Elliott et al. (2005) to determine critical velocities and shear stresses. The measurements were carried out by positioning a flume in the stream above the area of material to be tested. The water velocity over the material was gradually increased and the suspended sediment concentration and bed levels were measured to determine when erosion was occurring. The lowest velocity tested was 0.15 m/s and some sediment was eroded at this velocity in all tests. This led Elliott et al. (2005) to conclude that the critical velocity was between zero and 0.14 m/s. A critical velocity of 0.14 m/s is consistent with the critical velocity for unconsolidated cohesive sediments (Figure 2) and suggests that this method of determining critical velocity was applied to unconsolidated or disturbed material. The relatively low critical velocity derived by this method of testing is also inconsistent with values obtained by the jet tests described in the following paragraph and most shear tests listed in Table 1.

However, Elliott et al. (2005) also carried out a series of jet tests in a wider range of streams. In these tests, they directed a water jet at the sediment surface and measured the amount of erosion that occurred. From these measurements, they calculated a critical shear stress that varied from 4 N/m^2 to 72 N/m^2 (Figure 3). The average critical shear stress was 35.5 N/m^2 ± 19.4 S.D. This range of critical shear stresses is in agreement with the allowable shear stresses of 1 to 31 N/m^2 derived from U.S.S.R. data (Ven Te Chow 1959).

Figure 3

Critical shear stress measured by jet tests in Auckland streams/locations, showing allowable limits from Chow (1959).



Elliott et al. (2005) related critical shear stresses derived from jet tests to the properties of Auckland soil types. They found that the weakest soils (average 12 ± 9 S.D. N/m^2) were in the Puketoka formation and were described as silt rather than clay. There was little difference between the other soil types (Waitemata average 33 ± 19 S.D. N/m^2; Holocene average 48 ± 16 S.D. N/m2; Pumiceous average 47 ± 15 S.D.

N/m^2) and these were variously described as hard, compacted and clayey.

The measurements of soils on the bed, banks and surroundings of the Auckland streams carried out by Elliott et al. (2005) indicate that the clay content was sufficient to classify most streams as being in cohesive soils. The critical shear stress derived from jet tests relates to the strength of the material at the base of the scour hole, rather than to the surface material that is often less dense. The critical shear stress in the flume tests is the shear stress that begins to erode the surface material and the experiments determined a critical velocity of approximately 0.15 m/s. This is the typical of unconsolidated sediments, and it is possible that the surface sediments in the streams were unconsolidated or that they were disturbed during the experiments. In the flume tests of bank sediment, the sample was cut from the bank in slabs and these were placed in a tray under the flume. The jet experiments of Elliott et al. (2005) on undisturbed soils gave critical shear stresses that agree with recommended design shear stresses for cohesive soils. However, as expected there was considerable variation. Silty soils, rather than clayey soils, appeared to have the lowest strengths, but there were no clear distinctions between geological classifications.

3.2 Sediment transport/erosion formulae – literature review

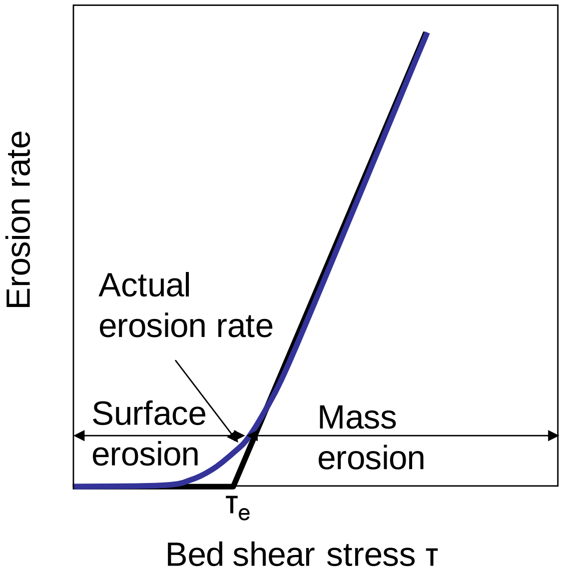
3.2.1 Non-cohesive sediment

Numerous empirical sediment transport formulae (bed load formulae) have been derived for non-cohesive sediments and the predicted sediment transport rates vary considerably, as does the amount of scatter in field measurements (Henderson 1966). The Einstein bed load formula is one of the best known for non-cohesive sediment and for high values of shear stress it predicts that bed load transport increases to the cube of shear stress (Henderson 1966).

3.2.2 Cohesive sediment

Cohesive sediment is either eroded from the bed in particles (surface or particle erosion) or in layers or blocks (mass erosion) that were formed in the deposition process or formed naturally in the soil matrix by micro-fissures due to various phenomena including compression and extension (Raudkivi 1990; University of California 2006). The surface erosion occurs when the applied shear stress exceeds a certain critical shear stress, and mass erosion happens when the applied shear stress exceeds the bulk strength of the sediment and is generally thought to be the most significant form of erosion for river channels (BCHF 2001). Krone (1999) describes this process: "the surface erosion rate increases linearly with shear stress until the pressure fluctuations and surface shear become sufficient to dislodge chunks of the bed surface. Further increase in shear stress causes rapid disintegration of a remoulded sample". Figure 4 shows an idealised relationship between erosion rate and shear stress, with surface erosion at low shear stress and mass erosion when the shear strength of the bed material is exceeded. It is possible to model this type of relationship with a linear relationship through the mass erosion component of the curve, a piecewise linear relationship through the surface and mass erosion components, or an exponential curve, and all three forms have been used (Table 2).

Figure 4
Idealised relationship between erosion rate and bed shear stress in cohesive material from Langendoen (2000).



Partheniades (1965) found that the surface erosion rate is a linear function of the dimensionless excess shear stress. His studies were carried out using San Francisco mud with a dry density of 614 kg/m^3, maximum velocity of 0.71 m/s, and shear stress of 1.33 N/m^2. Thorn and Parsons (1980) tested unconsolidated estuarine sediments (dry density up to 200 kg/m^3) up to shear stresses of 1.5 N/m^2. Ariathurai and Arulanandan (1978) found that remoulded cohesive sediments (uniform material consolidated at a pressure of 10,000 kg/m^2) showed a linear relationship between shear stress and erosion rate, but a few undisturbed samples showed a non-linear relationship that they attributed to the armouring of the surface. They tested over 200 samples up to a shear stress of 6 N/m^2. Parchure and Mehta (1985) examined soft cohesive estuarine deposits up to shear stresses of 0.4 N/m^2 and found that the erosion rate increased exponentially with excess shear stress. The dry density of the sediment varied from 150 kg/m^3 at the surface to about 400 kg/m^3 at 5 cm. In a review of estuarine sediment erosion, Mehta (1986) concluded that the exponential relation was valid for partly consolidated beds and the linear relationship was valid for fully consolidated beds.

Navarro (2004) measured erosion rates in a flume that was capable of generating a maximum velocity of 1.7 m/s and shear stresses of up to 20 N/m^2. He observed that erosion occurred with two mechanisms. At low shear stresses, there was surface erosion where single particles were dislodged over the entire bed. At high shear stresses, mass erosion occurred where the material failed along a plane causing very high erosion rates. These two mechanisms and their relative erosion rates are shown in Table 2, where low shear stress equations have M_1 = 0.007 to 0.117 kg/m^2/s and high shear stress equations have M_2 = 0.41 to 2.45 kg/m^2/s. Navarro (2004) converted the equations to the form $E = M_3 (\tau - \tau_c)^2$ and showed that the coefficient M_3 increased with particle size and decreased with the proportion of silt and clay in the samples and that these two variables explained 79 per cent of the variance in the coefficient.

Table 2

Some equations used in quantifying erosion rates of cohesive sediments.

Equation*	Proposed by
$E = M \dfrac{(\tau - \tau_c)}{\tau_c}$	Partheniades (1965), Ariathurai and Arulanandan (1978)
$E = M_3 (\tau - \tau_c)$	Thorn and Parsons (1980)
$E = M_1 \dfrac{(\tau - \tau_c)}{\tau_c}$ for $\tau < \tau_{c2}$ $E = M_2 \dfrac{(\tau - \tau_{c2})}{\tau_{c2}}$ for $\tau \geq \tau_{c2}$	Otsubo and Muraoka (1988), Krone (1999), Navarro (2004)
$E = E_c e^{\alpha\left(\frac{\tau - \tau_c}{\tau_c}\right)}$	Navarro (2004)
Equation*	Proposed By
$E = E_0 e^{\alpha (\tau - \tau_c)^{1/2}}$	Parchure and Mehta (1985)
$E = A\rho^m \tau^n$	Roberts et al. (1998)
$E = \dfrac{a_0}{T_d^m}\left(\dfrac{\tau - \tau_c}{\tau_c}\right)^n$	Hawley (1991)
$E = \mathrm{B}e^{\alpha (\tau_b - \tau_s)}$	Raudkivi and Hutchinson (1974)

* where *E* is the erosion rate (kg/m^2/s), *M*, M_1, M_2 and M_3 are constants (M = 0.005 to 0.015 kg/m^2/s, M_1 = 0.007 to 0.117 kg/m^2/s, M_2 = 0.41 to 2.45 kg/m^2/s (Navarro 2004), M_3 = 1.39 to 2.63x10^{-3} kg/N/s), τ and τ_c are the bottom shear stress and the critical bottom shear stress (N/m^2), E_0 x 10^5= 0.04 to 3.2 g/cm^2/min, α = 4.2 to 25.6 m/N$^{1/2}$ (Parchure and Mehta 1985), E_c = 0.0019 kg/m^2/s, α = 0.63 to 28.7 m/N$^{1/2}$ (Navarro 2004).

BCHF (2001) reviewed erosion rate formulae by Croad (1983), Roberts et al. (1998), and Partheniades (1965). Croad's (1983) formulae predicted an erosion rate that increased exponentially with shear stress and is based on a constant (number of molecular bonds) that is difficult to determine. Roberts et al. (1998) derived a formula that showed that the erosion rate of fine quartz particles increased with shear stress to the power of about 2. Krone (1999) describes the quartz particles used by Roberts et

[2] E is erosion rate, M a constant and τ and τ_c are the bottom shear stress and the critical bottom shear stress.

al. (1998) as weakly cohesive. BCHF (2001) considered that the constants derived in Partheniades 1965 study were not applicable because they related to particle erosion only and were developed for unconsolidated marine or lacustrine deposits.

Various equations quantifying erosion rate as a function of hydraulic shear stress have been developed, and a selection are shown in Table 2.

The linear relationship of Ariathurai and Arulanandan (1978) is the same as that proposed by Thorn and Parsons (1980) if the constant M_3 is replaced by M divided by the critical shear stress. The formulation suggested by Ariathurai and Arulanandan (1978) is commonly used to model erosion in streams with compacted cohesive soils (eg, Langendoen 2000; Willis and Krishnappan 2004), but the Thorn and Parsons' (1980) formula is more appropriate when the critical shear stress is zero or near zero, because there is no division by a zero or near zero number and the formula reduces to a constant times the shear stress. The Parchure and Mehta (1985) formulation is derived for soft partially consolidated beds. Some investigators, primarily working in the marine or lake environments, have studied erosion rates at low stresses in relatively unconsolidated cohesive marine or lacustrine sediment and have found low or no critical shear stress (eg, Partheniades 1965; Thorn and Parsons 1980). Studies at high shear stresses in more consolidated sediments have usually been carried out by engineers interested in erosion for scour or prediction of channel stability, and have shown relatively high critical shear stresses with linear relationships between erosion rate and excess shear stress. All erosion formulae include coefficients that are related to the material properties, such as mineral composition, organic material, salinity, dry density, temperature, pH value, and the Sodium Absorption Ratio (SAR).

From this review, it appears that the relationship between shear stress and erosion rate in cohesive consolidated soils can be approximated as linear, and that significant departures from linearity are caused by the properties of the soil (ie, low cohesive strength) or the degree of consolidation of sediment deposits varying with depth. The latter case usually occurs when erosion rates are measured by gradually increasing the applied shear stress over naturally deposited material, and the erosion rate per N/m^2 decreases with applied shear stress as the material erodes and the density increases.

One example of an applied erosion model in stream channels has been developed by the U.S. National Sedimentation Laboratory. The CONservational Channel Evolution and Pollutant Transport System (CONCEPTS) computer model (Langendoen 2000) simulates the evolution of incised streams and evaluates the long-term impact of rehabilitation measures to stabilize stream systems and reduce sediment yield. For cohesive bed material, erosion rate is calculated following the excess shear stress approach of Ariathurai and Arulanandan (1978).

One of the few studies that analyses actual stream erosion rates was carried out by Julian and Torres (2006). They compared measured erosion in cohesive soils of four stream reaches with various hydraulic erosion parameters, such as shear stress excess at peak-flow, time above critical shear stress, and shear stress excess integrated over time. They describe a method of calculating bank shear stress from channel shape and slope and give a formula for critical shear stress based on silt/clay percentage *sc*:

$$\tau_c = 0.1 + 0.1779sc + 0.0028sc^2 - 2.34\times10^{-5}sc^3$$

They also give a table of coefficients to multiply τ_c by to allow for bank vegetation. These coefficients vary from 1.97 for grassy banks to 5.4 for sparse trees to 19.2 for dense trees.

Julian and Torres (2006) concluded that the best predictor of erosion in moderately cohesive soils was bank shear stress excess $\tau - \tau_c$ at peak-flow.

3.2.3 Erosion rates for Auckland cohesive soils

Elliott et al. (2005) carried out a series of flume measurements in five Auckland streams/locations where they increased the water velocity flowing over a sample of the bank or bed material. The velocities varied from 0.15 m/s to 0.75 m/s and these were converted to shear stress by Debnath et al. (2007) assuming that the shear stress (N/m^2) was 1.6 times the square of velocity (m/s). The constant 1.6 was determined by measurement of the vertical velocity profile above soil samples. The rate that sediment was eroded into suspension (ie, suspended sediment concentration times the velocity $kg/m^2/s$) was measured, as well as the total sediment load from measurements of the eroded surface. Total sediment load varied erratically with shear stress (Figure 5), whereas re-suspension increased with shear stress (Figure 6). Linear relationships through zero were fitted to the erosion rate/shear stress data to determine average erosion rate constants in Table 3.

In eight of the 10 tests with bank sediment, there was no significant linear relationship ($P > 0.05$) between suspended sediment concentration and shear stress. In two cases, there was a significant positive relationship ($P < 0.012$). Examination of the graphical relationships showed that there was a tendency for suspended sediment concentration to increase with shear stress and velocity in six of the 10 tests, with the rate of increase decreasing at higher shear stresses. This result suggests that the suspended sediment concentration resulting from water flowing over cohesive sediment does not increase markedly with shear stress. The total flux of sediment in suspension varied linearly with shear stress, as shown in Figure 6, but gave a better linear relationship than with velocity in only five of the 10 cases. Mimura (1989) describes similar behaviours in a review of Japanese studies of cohesive sediment transport in a marine environment. In those studies, suspended sediment concentration was relatively constant at shear stresses greater than 0.1 N/m^2 under tidal cycles and wave action.

The data in Figure 6 divide into two groups, one where the re-suspension rate (measured suspended sediment concentration) increases slowly with shear stress and other where it increases sharply. The sites with low re-suspension rates were the two Papakura sites, Oratia LB, and Nukumea LB. The physical characteristics of sediment at these locations were similar to those of the other locations.

Figure 6 also indicates that the critical shear stress was effectively zero, contrary to the results of the jet test that indicated a median critical shear stress of 33 N/m^2 (range 4 N/m^2 to 72 N/m^2 Figure 3). The maximum shear stress tested in the flume was 0.9

N/m², which is considerably lower than the critical shear stresses from the jet measurements.

Figure 5

Total bed erosion of stream banks in Auckland as a function of bed shear stress with data points connected by lines (left) and linear regression lines (right). Nukumea_RB is not shown because erosion rates at this site were an order of magnitude higher than at other locations.

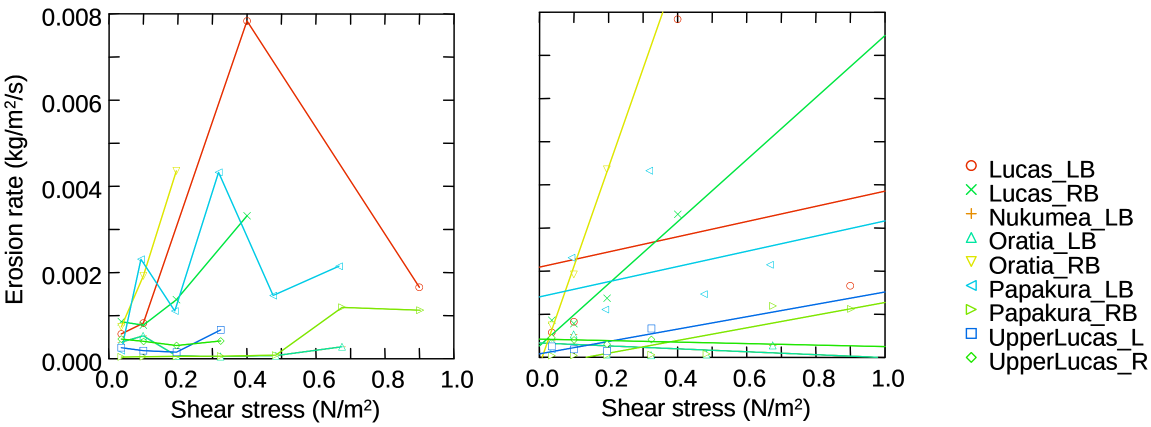
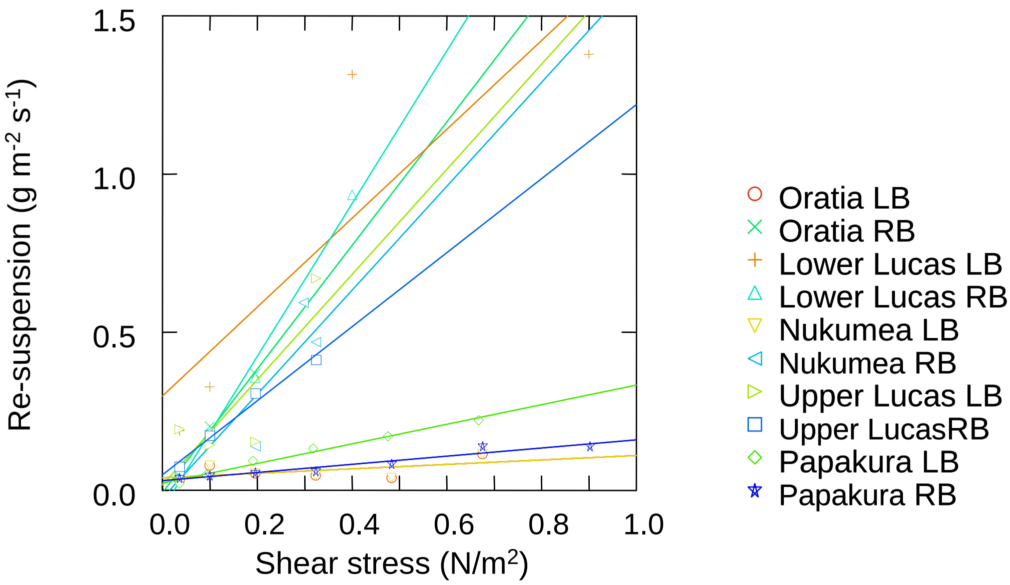


Figure 6

Re-suspension rate of stream banks in Auckland as a function of bed shear stress showing linear regression lines.



The erosion rates from the Auckland flume and jet tests were compared with some published rates (Table 3). These are presented in original units, g/m²/s and m/h, with the latter calculated assuming a dry density of 967 kg/m³ so that rates of erosion can be visualised in terms of the amount of material eroded from a stream in an hour and the realism of the predictions assessed from practical experience. The erosion rates correspond to the slope of the relationship between erosion rate and shear stress (ie, the coefficient M_3 in $E = M_3 (\tau - \tau_c)$. The erosion rates in Table 3 vary from a few millimetres per hour to metres per hour. Closer examination of test results from the Auckland jet tests, Navarro (2004) and Briaud et al. (1999) indicated that high rates corresponded to soils with weak cohesion. The very low erosion rates (< 10 mm/h) are for well-compacted clay soils.

Table 3

Measured erosion rates per N/m^2 in cohesive sediments.

Author(s)	Erosion rate		
	Author's units	g/m^2/s	m/h*
Shaikh et al. (1988)	0.3-0.8 N/m^2/min	0.5-1.4	0.002-0.005
Thorn and Parsons (1980)	0.158 kg/m^2/min	2.63	0.01
Ariathurai and Arulanandan (1978)	0.003-0.008 g/cm^2/min	0.5-0.8	0.002-0.003
Kelly and Gularte (1981)	0.0057-0.01 g/cm^2/s	57-100	0.21-0.37
Briaud et al. (1999)	0.0004-0.4 kg/m^2/s	0.4-400	0.001-1.5
Aberle et al. (2003)	0.003-0.087 kg/m^2/s	3-87	0.01-0.32
Navarro (2004)	0.05 kg/m^2/s (>20% clay)	50	0.19
Auckland streams flume tests Debnath et al. (2007)	0.004-0.2 kg/m^2/s	4-200	0.015-0.75
Auckland streams jet tests Elliott et al. (2005)	0.001-0.15 kg/m^2/s	1-150	0.003-0.56

* assuming a dry bulk density of 967 kg/m^3 as measured in Auckland streams by Elliott et al. (2005).

The review of cohesive sediment erosion formulae (Section 3.2.2) indicated that erosion rates increased linearly with excess shear stress where the sediment was cohesive and uniformly dense, as in the experiments of Ariathurai and Arulanandan (1978), and that the form of the relationship could be expressed as $E = M_3 (\tau - \tau_c)$. Although the measurements in naturally deposited sediments of Auckland streams showed variable relationships, they were not inconsistent with a linear relationship (Figures 5 and 6), especially the measurements of re-suspension. If potential erosion of Auckland streams is to be calculated using $E = M_3 (\tau - \tau_c)$, we must estimate values of the critical shear stress (τ_c) and erosion rate coefficient (M_3) and these can vary considerably between Auckland streams because of the different soil structures, particle sizes, mineralogy, and degree of consolidation.

The calculation of potential erosion rates from flow hydrographs, such as from storm water detention ponds of varying size, is more sensitive to critical shear stress than it is to the erosion rate coefficient. For example, Figure 7 shows three hypothetical shear stress hydrographs representing an unregulated hydrograph and hydrographs with moderate and high degrees of regulation where moderate regulation reduced peak stress by 50 per cent and high regulation reduced it by 66 per cent. For each of these hydrographs, the average shear stress over 19 hours is the same (4 N/m^2). Erosion was calculated for each hour and averaged over the 20 hour period for each of the "hydrographs" for three scenarios:

- Zero critical shear stress and erosion coefficient of 0.01 $kg/m^2/s$.
- 5 N/m^2 critical shear stress and erosion coefficient of 0.01 $kg/m^2/s$.
- 5 N/m^2 critical shear stress and erosion coefficient of 0.005 $kg/m^2/s$.

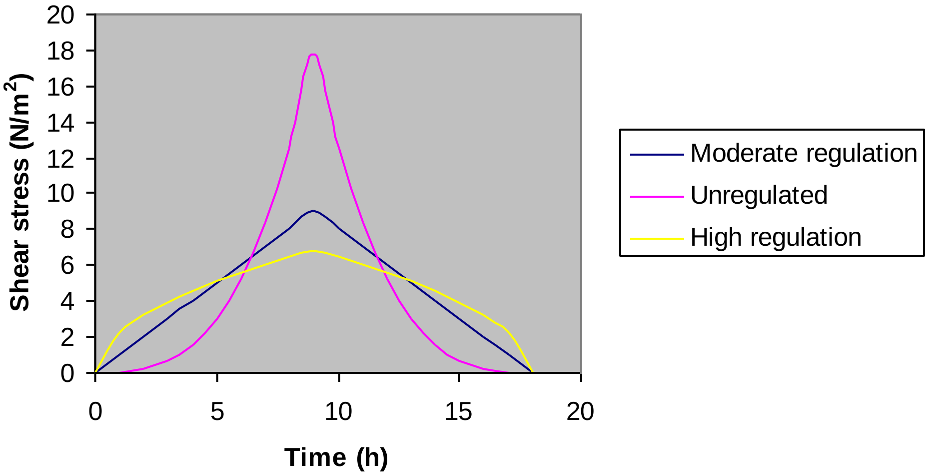
Table 4 shows the results of calculating the average erosion rate for each of these hydrographs and the three scenarios. With zero critical shear stress, the average erosion rate is the same for all three hydrographs. With a critical shear stress of 5 N/m^2, each degree of regulation reduces the average erosion rate by 50 per cent. If the erosion coefficient is reduced to 0.005, each degree of regulation also reduces the average erosion rate by 50 per cent. Thus, the erosion rate coefficient does not influence the comparison of flow regulation alternatives. Thus, the critical shear stress is the key parameter when comparing different hydrographs. The shear stress "hydrographs" in this example could be converted to discharge hydrographs using relationships between shear stress and discharge (as shown in Section 3.3). Because the flow/shear stress relationships are close to linear, the result of comparing flow hydrographs would be essentially the same as comparing shear stress hydrographs, as in this example, with critical shear stress having more effect on the relative change in erosion rate than the erosion rate coefficient.

Table 4

Average erosion rates over 19 h for the hypothetical shear stress "hydrographs" shown in Figure 7 for zero critical shear stress and 5 N/m^2 critical shear stress.

Hydrograph	Critical shear stress (N/m^2)	Erosion rate coefficient $kg/m^2/s$	Average erosion rate $kg/m^2/h$
Unregulated	0	0.01	153
Moderate regulation	0	0.01	153
High regulation	0	0.01	153
Unregulated	5	0.01	70
Moderate regulation	5	0.01	32
High regulation	5	0.01	16
Unregulated	5	0.005	35
Moderate regulation	5	0.005	16
High regulation	5	0.005	8

Figure 7
Hypothetical shear stress "hydrographs" with varying degrees of flow regulation.



The median critical shear stress for Auckland soils was 33 N/m^2 and ranged from 4 N/m^2 to 72 N/m^2 (Elliott et al. 2005). It is likely that soil conditions and critical shear stress will vary along the length of a stream and that local channel morphology will have adjusted accordingly, with channel width and slope varying inversely with bank strength. We suggest using the median value of critical shear stress (33 N/m^2). Calculated shear stresses at near bank full flow at the flume sites described in Elliott et al. (2005) are in the range 40-105 N/m^2. However, field verification should be undertaken in streams by calculating the shear stress/flow relationships over stable stream reaches following the methods used by Julian and Torres (2006). Shear stresses during past high-flow events, such as the channel forming (bank full) discharge, can then be used as a guide to the critical shear stress. Shear stress during high-flow events can also be compared to soil critical shear stress (carried out along stream banks using jet tests) and assessments of channel morphology and bank vegetation to determine the degree to which these factors influence erosion. This analysis assumed a linear relationship between excess shear stress and erosion rate. If the relationship is not linear, as shown in Figure 4, then erosion will be greatest in the scenario with the highest shear stress, as Julian and Torres (2006) found in their study.

3.3 Calculation of shear stress in streams

As described earlier (Section 3.1), shear stress is difficult to measure directly and in streams it is usually inferred from the water depth and water surface slope, ie, $\tau = \gamma RS$. This is a measure of the total shear stress on the channel and does not take into account local effects caused by bends, minor channel irregularities, flow obstructions such as plants. Also, shear stress varies across a cross-section. Hence, using this formula and other simplified hydraulics equations such as Manning's equation to estimate shear stress uses an idealised and simplified representation of the actual hydraulics.

Prediction of shear stresses using the formula above therefore requires an estimation of hydraulic radius R and slope S over the full range of flows. At high-flows, local variations in water surface slope are often drowned and the water surface slope is relatively uniform and parallel to the average bed slope. It is more difficult to predict how the hydraulic radius or water depth varies with flow because channel roughness varies with flow. Usually, roughness is high at low flows and decreases as flows increase to about bank full, then increases again as flow goes over bank. Uniform flow equations, such as Manning's equation (Section 3.1), are often used to predict water depth from flow, assuming slope and roughness. Slope is often measured in the field, but Manning's n is often assessed visually by comparison with photographs. However, it is preferable to use calculated values of Manning's n and its variation with discharge to predict shear stress. This can be done from rating curves, as described below. This is a standard procedure in instream habitat methods (IFIM, RHYHABSIM) and was also used to estimate stream shear stress in Julian and Torres' (2006) study of erosion.

The relationships between shear stress and flow can be calculated from the rating curves assuming a) that water surface slope is constant and Manning's n is calculated from the rating curve and cross-section, and b) that Manning's n is constant and that the water surface slope is calculated from the rating curve and cross-section, as shown in Table 5. This procedure can also be carried out without any rating curves, with assumed values of slope and Manning's n, and with hydraulic radius determined by a trial and error solution of the uniform flow equation.

Application of this procedure to a few example streams and rating curves (Figure 8) shows that the rate of increase of shear stress with discharge depends on the stream and hydraulic assumptions that are made. As noted earlier, the assumption that the slope is constant (at high-flows) and that Manning's n varies with discharge is probably the most accurate for the estimation of erosion, and in both examples, this assumption results in much lower shear stress than assuming constant roughness or uniform flow. If rating curves are not available, the variation of Manning's n with discharge must be estimated and the streams shown in Hicks and Mason (1991) provide a guide.

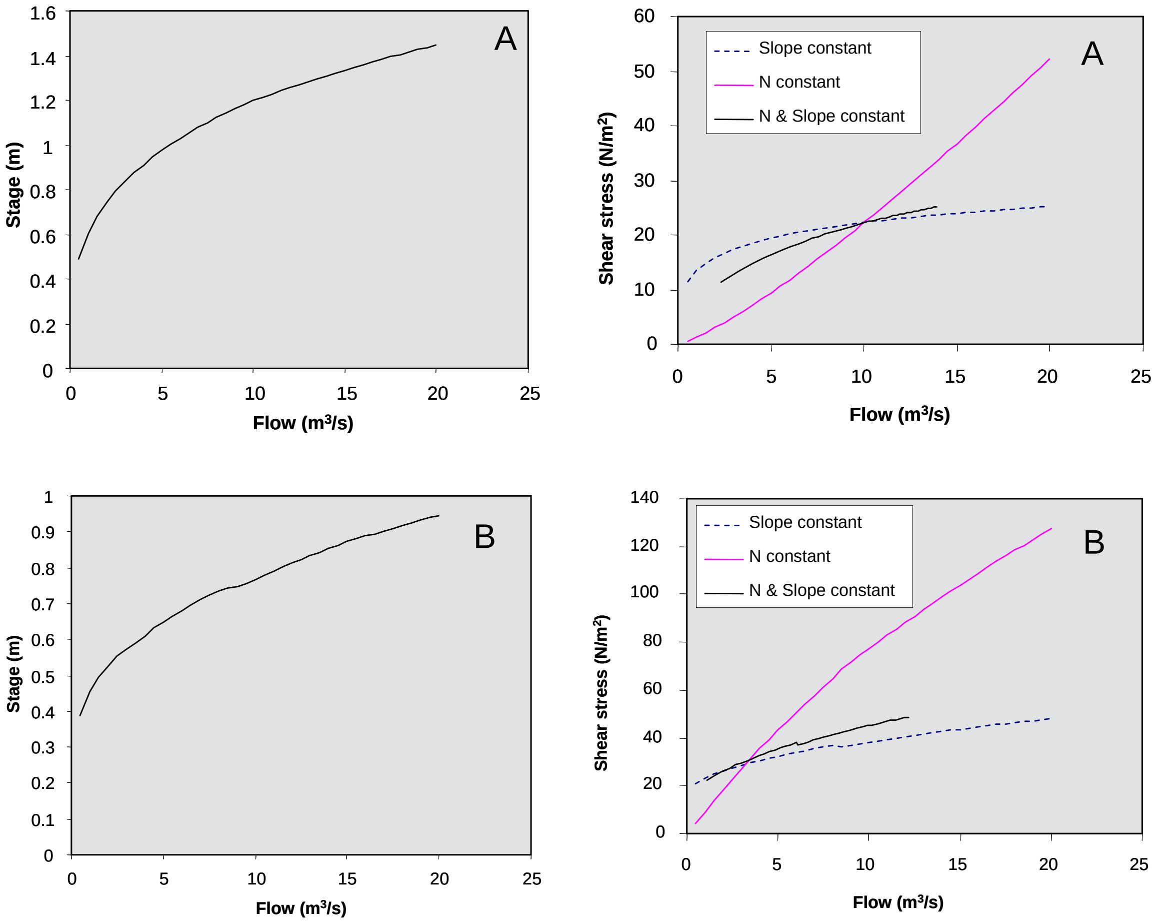
Table 5

Example of calculation of shear stress using a) an assumed slope S = 0.003 and b) an assumed Manning's n = 0.02.

Stage (m)	Flow (Q m^3/s)	Area (A m^2)	Wetted Perimeter (P m)	Hydraulic radius (R m)	Velocity (V m/s)	n variation with constant slope	Shear stress (N/m^2) with constant slope	Slope (S) variation with constant n	Shear stress (N/m^2) with n constant
From rating	From rating	From cross-section	From cross-section	A/P	= Q/A	$\frac{1}{V} \times R^{2/3} \times 0.003^{1/2}$	$999.1 \times 9.81 \left(\frac{V \times N}{R^{1/6}} \right)^2$	$\left(\frac{V \times 0.02}{R^{2/3}} \right)^2$	$999.1 \times 9.81 RS$
0.49	0.5	1.585	4.068	0.3896	0.315	0.09262	11.456	0.00013988	0.534
0.602	1	2.001	4.342	0.4608	0.500	0.06539	13.551	0.00028064	1.268

Figure 8

Examples of rating curves (left) for reaches in two Auckland streams (A Hoteo t and B Alexandra) and relationships between shear stress and flow (right) calculated from the rating curves assuming a) that water surface slope is constant and Manning's n calculated from the rating curve and cross-section (dashed line), b) that Manning's n is constant and that the water surface slope is calculated from the rating curve and cross-section (solid blue line), and c) assuming uniform flow with Manning's n and slope constant (solid black line).



4 Conclusion

The literature review of erosion in cohesive soils indicated that for compacted clay soils, the erosion rate is linearly related to excess shear stress. Departures from this relationship occur when soil properties vary with depth or when the characteristic of the soil surface changes, such as by armouring. The relationships between shear stress and erosion rates in the NIWA Auckland study (Elliott et al. 2005; Debnath et al. 2007) tended to support a linear relationship, although exponential relationships were better in some cases. The NIWA studies also showed erosion rates varying from a few millimetres per hour to about 0.5 m per hour depending in the cohesive strength of the soil. These erosion rates were consistent with other reported studies. The critical shear stress of Auckland soils as measured by jet tests (Elliott et al. 2005) were also consistent with reported studies, but were not consistent with the flume tests (Elliott et al. 2005; Debnath et al. 2007) that showed low or zero critical shear stresses. The NIWA studies showed that the critical shear stress and erosion rates were very variable and this is consistent with international experience of cohesive soils. The variability relates to differences in soil mineralogy, texture, degree of compaction, and organic matter content. However, channel sizes will have adjusted to natural variations in soil properties, with wide sections where the soil is weak and narrow and/or steep sections where the soil strength is high. Thus, because the channel characteristics vary with soil strength, it is possible to use an average value of critical shear stress and average cross-section shape for design purposes.

The method of estimating shear stress in streams is as important as the erosion equations. We recommend that relationships between shear stress and flow be derived from measured relationships between flow and stage. If uniform flow equations are used to predict water levels, then some variation of Manning's n with flow should be incorporated in the analysis.

We recommend that an erosion equation for cohesive Auckland streams follows that most commonly used in modelling studies $E = M_3 (\tau - \tau_c)$. The parameters in this equation will apply to both pre- and post-development scenarios. An assessment of erosion rates resulting from urbanisation should take account of both the erosion threshold (τ_c) and the extent to which this threshold is exceeded. We recommend that site specific studies be carried out to determine relationships between soil properties, as determined by the relatively simple jet tests, channel morphology, bank vegetation, and total shear stress at the channel forming (bank full) discharge. In particular, shear stress/flow relationships can be calculated for stable stream reaches and shear stresses during past high-flow events, such as the channel forming (bank full) discharge, used as a guide to the critical shear stress (eg, Julian and Torres 2006). If specific parameters are not developed for a stream, we suggest using the median critical shear stress (c. 33 N/m^2) and a value of 0.005 to 0.01 $kg/m^2/s$ for the coefficient M_3.

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